Michelle Tieu

Honors Differential Equations

I. Intro (1,2)

Blood plays a crucial role in the function of the human body and the maintenance of homeostasis. Homeostasis is when the body resists changes in order to maintain internal stability. For example, when body temperature drops below a certain level a person might start shivering, or when somebody starts exercising and depleting the oxygen in their blood at a faster rate than when resting, they will begin to breath faster and their heart rate will increase in order to distribute more oxygen throughout the body. (1)

The circulatory system is responsible for transporting oxygen to cells throughout the body, regulating temperature, transporting nutrients and cellular wastes, clotting, and other functions. Due to this, blood is a complex fluid that has specific qualities in order to fulfill all of these needs. Blood is composed of erythrocytes, leukocytes, and platelets. Erythrocytes are red blood cells that contain the hemoglobin protein that carries up to four oxygen molecules. This component of blood is what is responsible for gas exchange throughout the body. Additionally, leukocytes are the white blood cells that are responsible for immune functions and platelets are the what allow blood to coagulate. These three components are suspended in the liquid portion of blood called plasma. (2)

II. Circulatory System (4,16,1)

In order for blood to be distributed throughout the body, it follows the circulatory system. Starting at the heart, blood is pumped to the lungs in order to exchange carbon dioxide in the blood for oxygen, then it returns to the heart. From there it is pumped throughout the body through arteries, arterioles, capillaries, venules, veins, and then back through the heart. The difference between arteries and veins is that the blood flowing in arteries is moving away from the heart while the blood in veins is returning to the heart. These large blood vessels mostly function as pathways for blood to be transported to other parts of the body. (4)

The majority of gases, nutrients, and cellular wastes are exchanged at the capillary level. For each successive level of vessels starting at the arteries, smaller vessels branch out until they become capillaries. The total cross-sectional area of the capillaries is about 6000 cm^2 versus arteries that have a total surface area of about 40 cm^2 (16). While this number may seem large in comparison, capillaries have to be heavily abundant throughout the body otherwise certain tissues would not get the nutrients that they need. The branching network ensures that each cell will be close enough to a blood source. Eventually they join back together to form veins and direct blood back to the heart.

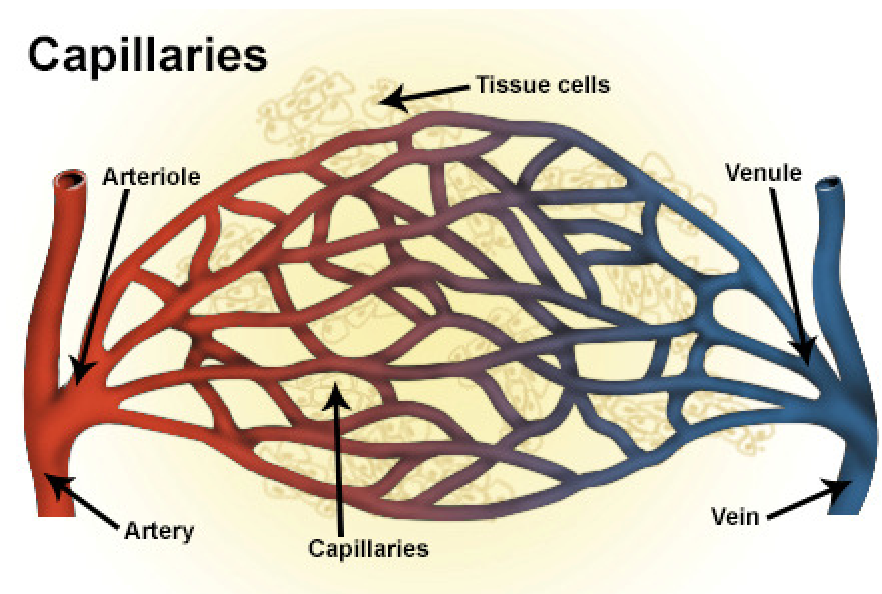

Figure 1 shows a basic model of how blood vessels change throughout the circulatory system (4).

One of the most important structural characteristics of arteries is that they are highly elastic. This allows the diameter, and thus localized blood volume, of the arteries to increase immediately after the heart pumps, thus maintaining a more stable pressure in these vessels. As a result, the amount of blood circulated throughout the body from each beat of the heart is maximized and the heart does not have to work excessively hard. For example, if arteries were not elastic, blood pressure would be significantly higher after each heartbeat, but not as much blood would be pumped through the vessels. The heart would have to compensate by pumping faster so that cells receive the oxygen they need. (1)

III. Blood Physics (1,4,8,9)

Even though capillaries have the smallest diameter of about 5-10 micrometers compared to arteries with diameters that can be larger than 10 millimeters and veins that range in diameter from 1 up to 15 millimeters, they have lower pressure. This is due to the vast branching network of the capillaries. Arteries contain about 10% of total blood volume while capillaries contain about 15%. The rest of blood volume, about 70%, is contained in the veins. This pressure gradient is what facilitates the flow of blood throughout the entire body. Due to the increase in volume of all of the capillaries in relation to the arteries, resistance of blood flow decreases. As the volume gradually increases in the capillaries, the pressure decreases, thus velocity decreases. This gives cells and blood enough time to transfer nutrients, wastes, and oxygen through diffusion. Velocity and pressure decrease even more as blood moves from capillaries to veins. Since pressure in veins is so low and blood has to work against gravity in the lower extremities, some veins have valves that prevent the backflow of blood when the heart is at rest, and they ensure that blood continues to move in one direction. (1)(4)

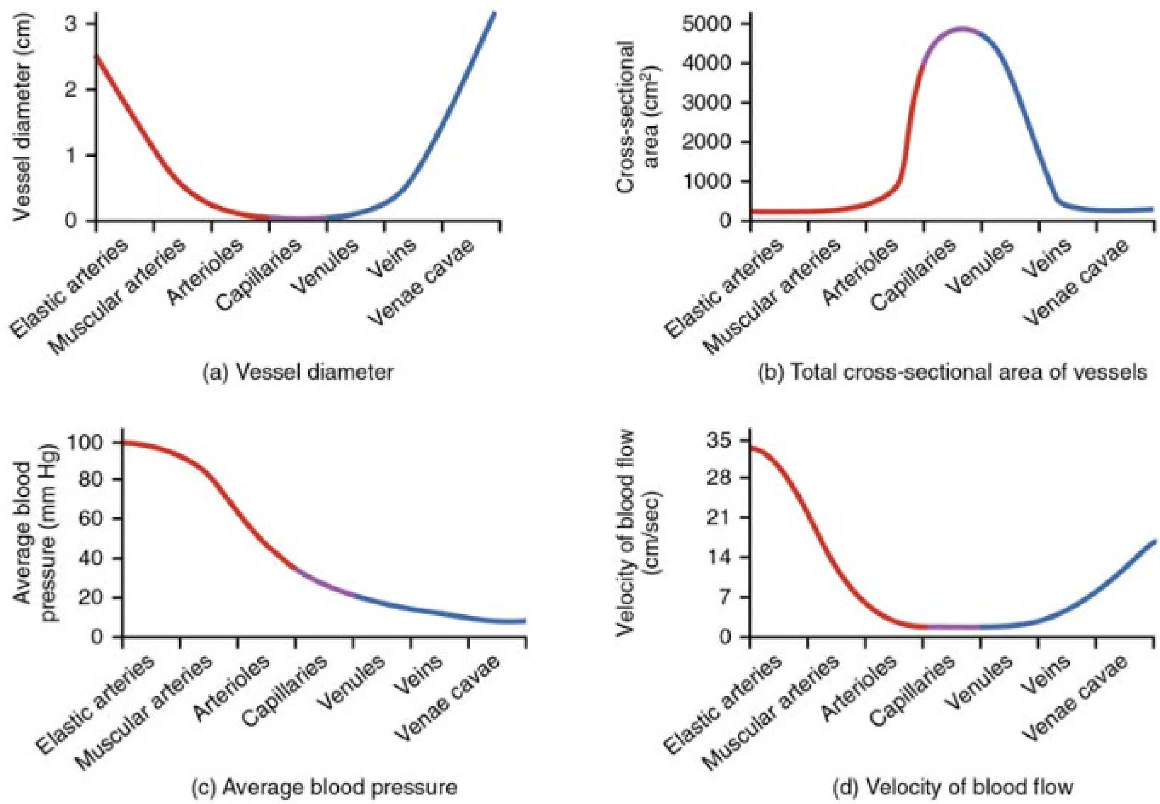

Figure 2: The relationship between (a) vessel diameter, (b) total cross-sectional area of the vessels, (c) average blood pressure, and (d) velocity of blood flow throughout the circulatory system is shown.

One vital factor that affects blood flow is viscosity. Viscosity is a property of fluids that is related to internal friction caused by fluid layers moving past each other. In blood, viscosity is proportional to resistance and inversely proportional to flow, meaning that high blood viscosity increases resistance and decreases blood flow. Blood viscosity is usually dependent on interactions between red blood cells and plasma proteins. More specifically, red blood cells experience friction when the move past each other in vessels. This is more significant than the effects of plasma proteins because cells are larger than the molecules found in plasma. Sometimes, red cells stick together and form small chains, further increasing the viscosity of blood. Since these molecules are given more time to interact at lower velocities, viscosity increases when velocity is low. Additionally, blood is slightly compressible since blood cells can compress and change shape to squeeze through capillaries. This makes blood a non-Newtonian fluid as opposed to a Newtonian fluid where "viscosity is independent of flow velocity" (8).

Another factor that can affect blood viscosity is temperature. When fluids are at a lower temperature, they become more viscous as the fluid has a lower kinetic energy and molecules do not readily move past each other, thus increasing friction between molecules. In the body, these changes in temperature are most evident in the extremities where velocity is already lowered due to vessel size. In these areas, changes in blood flow are even more significant and may have detrimental impacts on a person's health.

Lastly, a common factor that affects blood flow is simply the amount of water that is present in a person's blood. If they are dehydrated, there is more friction caused by blood contents restricted to a lower volume, leading to an increased viscosity. It has been found that, when a person is dehydrated, their blood viscosity when the heart pumps increased by about 9.4%; when the heart is momentarily at rest between beats, blood viscosity increases by about 12%. (9)

IV. History of Blood (6,5,7)

Until about the mid 1600's, people lacked significant knowledge about the functions of blood and ways that blood could be used to help sick people (6). Up until this point in modern medicine, most of what cultures knew about blood was that it was a life force, thus blood became symbolic of life and death (5). Naturally, blood played important roles in religion in many cultures. One of the most notable instances of a culture's use of blood was the Aztecs and their belief that blood sacrifice would appease the gods and cause good weather and bountiful crops. It is estimated that a few thousand humans and animals would be sacrificed each year for this purpose. The people were often prisoners of war and would have their beating heart cut from their chest as they lie strapped to a sacrificial stone (7).

V. History of Blood in Medicine (10,11)

Before the advancements of modern medicine and before doctors had an accurate understanding about how the body worked, one of the first treatments that they would resort to was bloodletting. The practice may have been made more common by Hippocrates' in about 460-370 BC and his idea that illness was caused by an imbalance of the four humors that made up the body. The four humors were blood, phlegm, black bile, and yellow bile (10). Naturally, if there was an imbalance of blood, the way to get bring it back to its natural levels was to participate in bloodletting. Even in medieval Europe, bloodletting was still being used as a treatment for a variety of illnesses such as smallpox and epilepsy. Often these procedures consisted of using a sharp pointed tool or a blade to nick an artery or vein, but leeches would sometimes be used.

Eventually, the church began to disprove of the practice, so barbers began performing these procedures for people in addition to their normal haircuts. People carried this practice on for many years. In fact, the death of George Washington has often been attributed to blood loss after he had about 5-7 pints of blood drained in less than 16 hours in response to a sore throat (11). Despite the medical knowledge that patients did not often benefit from bloodletting, people probably continued to live with the idea that this treatment was valid and turned to it when they felt ill. It is possible that people with certain diseases did benefit from this treatment, which may have helped people continue to believe that bloodletting was effective. One disease that still uses bloodletting as a treatment is hemochromatosis, which is the buildup of iron in the body. Periodic blood draws, are used to limit this accumulation. Now, blood is used much more practically. From blood transfusions to blood tests that allow people to be tested for hundreds of diseases, medicine has advanced to help thousands of people every day.

VI. History of Modeling Blood Flow (14,17,15,18,20)

Fluid modeling first began in ancient civilizations that had to solve various flow problems for various activities like sailing or planning irrigation systems. Between 285 and 212 B.C., Archimedes studied buoyancy and applied these laws to objects that could float and be partially submerged. He developed a screw pump that converted the potential energy of the fluid into mechanical energy of the screw. The pressure change caused by the turning of the screw caused the fluid to move against gravity. Its concept was revolutionary and allowed for low bodies of water to be transported into irrigation ditches that were higher up. His works were likely the early forms of differential calculus that would later be used for more advanced modeling. (14)(17)

The next major development for fluid modeling occurred when Leonardo da Vinci, who lived from 1452 to 1519, formed the equation for the conservation of mass in situations with one-dimensional steady flow. He also explored concepts of waves, jets, and situations with high and low drag. He had hundreds of accurate drawings about the behavior of fluids that helped future scientists with the mathematical modeling of these patterns (14). Additionally, when Isaac Newton, who lived between 1642 and 1727, defined what a Newtonian fluid was, he made it possible for many other mathematicians to derive other equations about fluid flow. His most famous work *Mathematical Principles of Natural Philosophy* was published in 1687 (18). This work stated the fundamental three laws of motion and defined the concept of gravity. These laws are the foundation of physics and are have many applications throughout physics.

Somebody who began to directly apply mathematical modeling to fluid flow in the body is Daniel Bernoulli, who lived from 1700 to 1782. When he began learning about fluid mechanics while trying to pursue the medical degree that his father expected from him, Bernoulli took advantage of the book *On the Movement of Heat and Blood in Animals* to further his research on the topic (15). The book compared the human heart to a pump that would cause blood to flow like a fluid through the circulatory system. Using the principles of conservation of momentum and conservation of energy, Bernoulli was able to explain the inverse relationship between pressure and velocity when the streamline is decreased. Bernoulli discovered how to measure blood pressure and explained that a "moving fluid exchanges its kinetic energy for *pressure*", eventually deriving the equation $\mathbf{p + \frac{1}{2}\rho u + \rho g h = constant}$, where "p" represents pressure, "ρ" represents the density of the fluid, and "u" represents it velocity (19). These concepts were explained in 1738 when he published his book *Hydrodynamica* (20). There are many restrictions on this model though: points 1 and 2 lie in streamline; fluid is incompressible; fluid has steady flow; and the fluid experiences no friction. If the fluid is incompressible, it has a constant volume, and thus constant density.

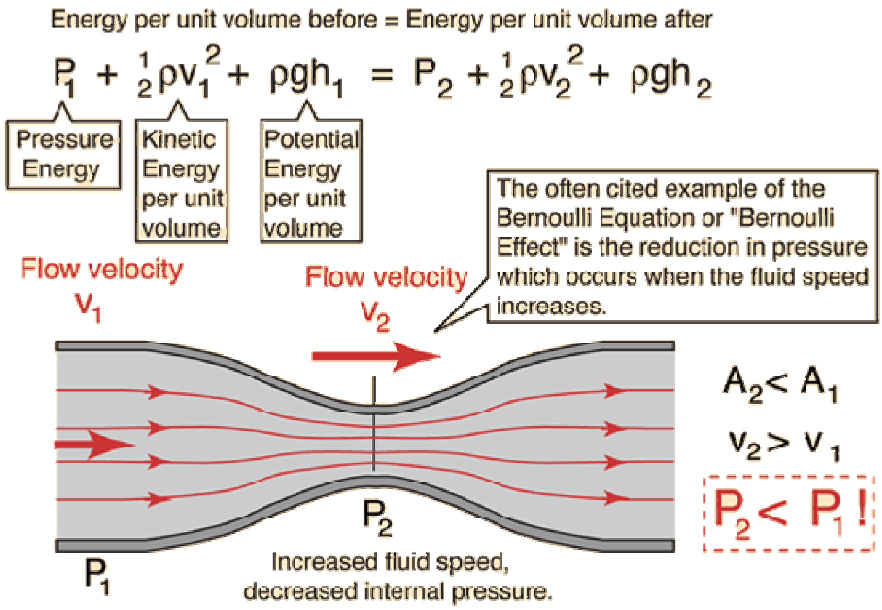

Figure 3 shows an application of Bernoulli's equation when vessel diameter is constricted. (20)

One model that has slightly less restriction in its model for fluid flow is Poiseuille's equation, which takes changes in viscosity into account. Jean-Louis-Marie Poiseuille lived from 1799 to 1869 and was a French physician who discovered the relationship between blood vessel diameter and flow velocity. He did this by using air pressure to force water through glass tubes that were the size of capillaries because, at the time, anticoagulants had not been created to keep blood from clotting outside the body and actual blood could not be used (22). His equation is for flow rate is given by $Q = \frac{\pi R^4}{8L\mu} P$ where "L" is the length of the vessel, "R" is the radius of the vessel, "μ" is the blood viscosity, and "P" is pressure (23).

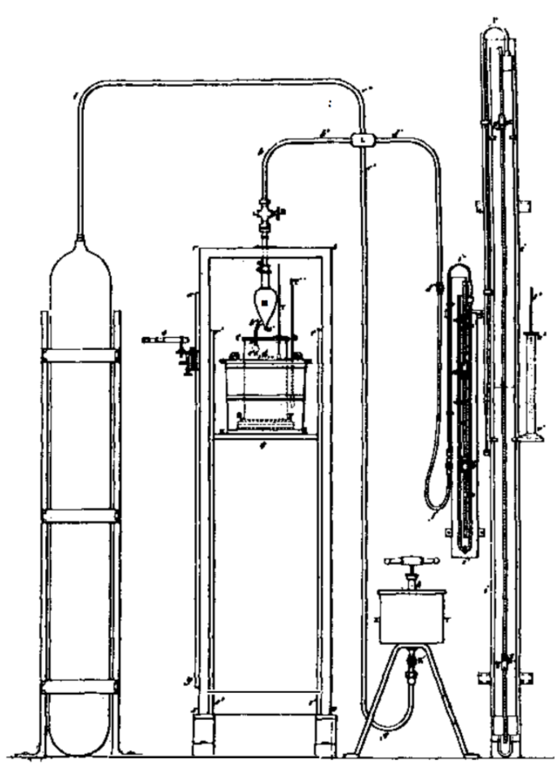

Figure 4 shows the contraption that Poiseuille used to study the *relationship between vessel diameter and blood flow (21).*

When studying for his doctorate, he published *The Force of the Aortic Heart* in 1828 and would later publish many more articles for science journals (21). Most of his works related to blood flow and nonturbulent fluids. Today, new combinations of these equations are being used to encompass more factors that could affect blood flow. In the future, it is hoped that a more accurate equation to model blood flow will aid in the treatment and research of different circulatory diseases that are fatal to many around the world.

VII. Derivation of Bernoulli's Equation (26)

Conservation of Mass:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0$$

Where: $\vec{\nabla} \cdot (\rho \vec{V}) \equiv \rho \vec{\nabla} \cdot \vec{V} + \vec{V} \cdot \vec{\nabla} \rho$

This can be plugged back into the top equation to give:

$$\frac{\partial \rho}{\partial t} + \vec{V} \cdot \vec{\nabla} \rho + \rho \vec{\nabla} \cdot \vec{V} = 0$$

We can use the derivative operator to modify the conservation of mass equation -

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \quad \rightarrow$$

Assuming that blood is incompressible: $\vec{\nabla} \rho = 0$

$$\frac{d\rho}{dt} + \rho \vec{\nabla} \cdot \vec{V} = 0$$

Conservation of Momentum:

Newton's Second Law can be used to further modify the conservation of momentum equation:

$$\sum \vec{F} = m\vec{a} \rightarrow$$

$$\sum \vec{F} = \frac{d}{dt}(m\vec{V}) \rightarrow$$

$$\oiiint_{\mathrm{v}} \frac{\partial(\rho\vec{V})}{\partial t} d\mathrm{v} + \oiint_S (\rho\vec{V}\cdot\vec{V}dS) = -\oiiint_{\mathrm{v}} \vec{\nabla} p d\mathrm{v} + \oiiint_{\mathrm{v}} \rho \vec{f} d\mathrm{v} + \oiiint_{\mathrm{v}} \vec{F}_{viscous} d\mathrm{v}$$

Where $\vec{V} = < u, v, w >$:

$$\oiiint_{\mathrm{v}} \frac{\partial(\rho u)}{\partial t} d\mathrm{v} + \oiint_S (\rho\vec{V}\cdot u dS) = -\oiiint_{\mathrm{v}} \frac{\partial p}{\partial x} d\mathrm{v} + \oiiint_{\mathrm{v}} \rho f_x d\mathrm{v} + \oiiint_{\mathrm{v}} (F_x)_{viscous} d\mathrm{v}$$

$$\oiiint_{\mathrm{v}} \frac{\partial(\rho v)}{\partial t} d\mathrm{v} + \oiint_S (\rho\vec{V}\cdot v dS) = -\oiiint_{\mathrm{v}} \frac{\partial p}{\partial y} d\mathrm{v} + \oiiint_{\mathrm{v}} \rho f_y d\mathrm{v} + \oiiint_{\mathrm{v}} (F_y)_{viscous} d\mathrm{v}$$

$$\oiiint_{\mathrm{v}} \frac{\partial(\rho w)}{\partial t} d\mathrm{v} + \oiint_S (\rho\vec{V}\cdot w dS) = -\oiiint_{\mathrm{v}} \frac{\partial p}{\partial z} d\mathrm{v} + \oiiint_{\mathrm{v}} \rho f_z d\mathrm{v} + \oiiint_{\mathrm{v}} (F_z)_{viscous} d\mathrm{v}$$

Momentum equations for 1-D motion can be simplified with some assumptions:

- $\frac{\partial}{\partial t} = 0$ because flow is steady
- $\rho g_x \simeq 0$ because only small changes in gravitational potential energy occur within blood vessels in small distances
- $F_{viscous} \approx \mathbf{0}$
- ρ is constant

These assumptions can be used to simplify the equation for conservation of momentum to Bernoulli's:

$$\rho\frac{\partial u}{\partial v} + \rho u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \rho g_x + (F_x)_{viscous}$$

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = 0$$

$$\frac{1}{2}\rho \frac{d(u^2)}{dx} + \frac{dp}{dx} = 0$$

$$\frac{1}{2}\rho u^2 + p = \text{constant} = p_o$$

In this equation, p_o is the stagnation pressure as the velocity approaches zero, or the max pressure that can be obtained.

Momentum equations for 2-D motion can be simplified with some assumptions:

- $\frac{\partial}{\partial t} = 0$ because flow is steady
- $\rho g_x \simeq 0$ because only small changes in gravitational potential energy occur within blood vessels in small distances
- ρ is constant

Where $\vec{V} = \langle u, v, w \rangle$:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \rho g_x \rightarrow \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = 0$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \rho g_y \rightarrow \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = 0$$

Another simplification that can be made is:

$$\tan(\theta) = \frac{dy}{dx} = \frac{v}{u} \rightarrow (u)(dy) = (v)(dx)$$

From here the x-momentum equation can be multiplied by dx in order to simplify it.

$$\rho u \frac{\partial u}{\partial x} dx + \rho v \frac{\partial u}{\partial y} dx + \frac{\partial p}{\partial x} dx = 0$$

$$\rho u(\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy)+\frac{\partial p}{\partial x}dx=0\rightarrow(\rho u)(du)+\frac{\partial p}{\partial x}dx=0$$

$$(\frac{1}{2}\rho)(d(u^2))+\frac{\partial p}{\partial x}dx=0$$

The same can be done with the y-momentum equation.

$$\rho u\frac{\partial v}{\partial x}dy+\rho v\frac{\partial v}{\partial y}dy+\frac{\partial p}{\partial y}dy=0$$

$$\rho u(\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy)+\frac{\partial p}{\partial y}dy=0\rightarrow(\rho v)(dv)+\frac{\partial p}{\partial y}dy=0$$

$$(\frac{1}{2}\rho)(d(v^2))+\frac{\partial p}{\partial y}dy=0$$

The derivations of the x and y momentum equations can be added to further simplify into the Bernoulli equation.

$$\frac{1}{2}\rho[d(u^2+v^2)]+\frac{\partial p}{\partial x}dx+\frac{\partial p}{\partial y}dy=0$$

$$\frac{1}{2}\rho[d(u^2+v^2)]+dp=0$$

$$\frac{1}{2}\rho V^2+p=p_o=\text{constant}$$

Again, in this equation, p_o is the stagnation pressure as the velocity approaches zero, or the max pressure that can be obtained.

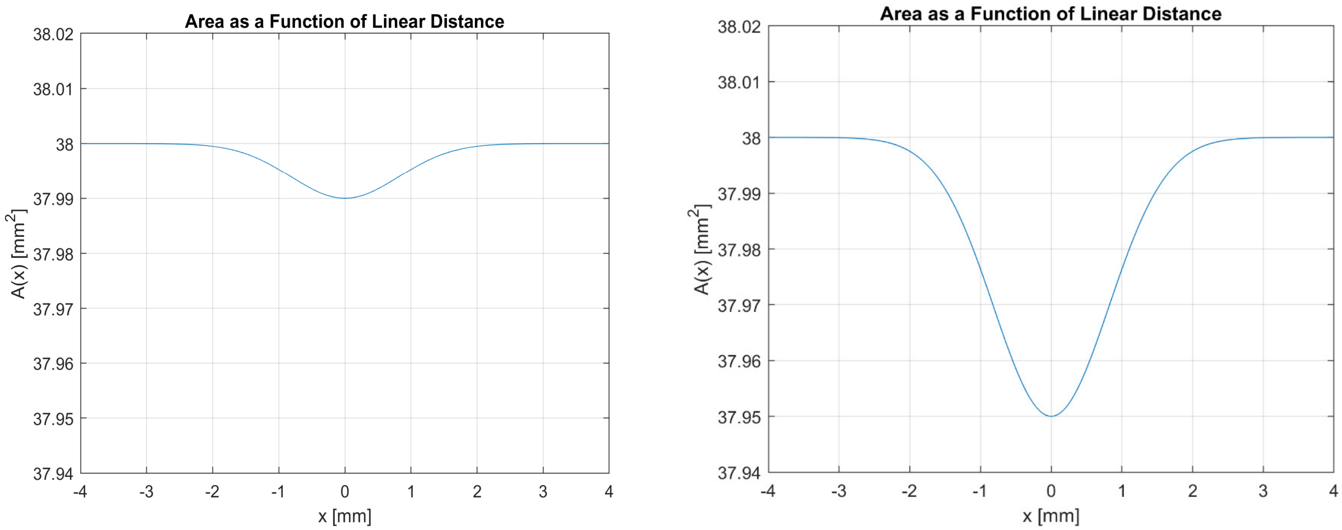

Figure 5 models the change in cross sectional area of a coronary artery with a 5% reduction amount and compares it to the change in cross sectional area of an artery with a 1% reduction.

VIII. MATLAB

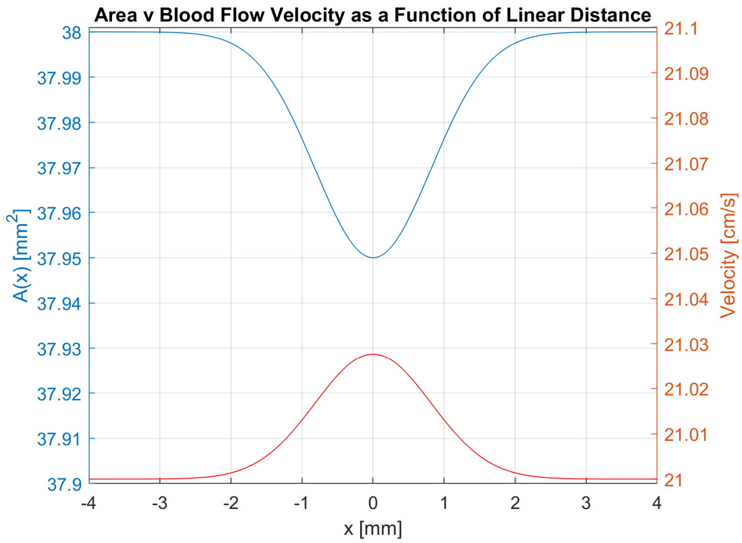

Figure 6 shows the change in velocity caused by a 5% area reduction in a coronary artery.

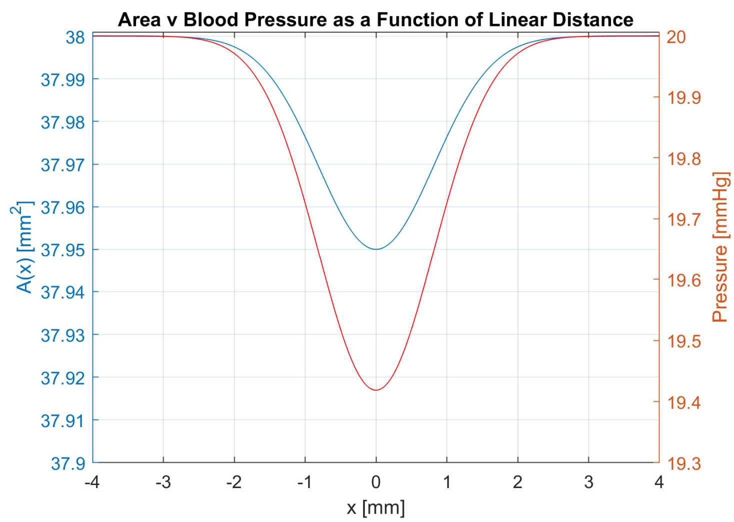

Figure 7 shows the change in blood pressure caused by a 5% area reduction in a coronary artery.

For this model, a coronary artery with a plaque formation will be modeled. As previously stated in the blood physics section, velocity increases in the vessel through the restricted area due to conservation of mass; more blood must flow through a smaller area so velocity increases. Pressure, however, decreases through the narrow section as verified by Bernoulli's equation. Additionally, the plaque formation was modeled to have normal distribution for simplicity. The initial pressure chosen for this model is a constant 20 mmHg, which is the average systolic blood pressure in the coronary arteries (27). Again, the beating of the heart was not accounted for in this model for simplicity.

IX. Problems with Blood Flow (1)

After the significance the structure of the circulatory system is explained, it is easy to understand how any factors that affect the diameter of a blood vessel greatly impact blood flow throughout the body. For example, one of the most common issues that people experience is the buildup of plaque in arteries. This restricts blood flow due to the narrowing of vessel walls in addition to making them less flexible. In extreme cases, the artery is blocked completely and cells do not receive the oxygen that they need to survive. Muscle death results if the blockage is not treated immediately. In order for all cells to receive the blood supply that they need to function properly, the heart will overcompensate by pumping more. This increases the amount of blood being delivered throughout the body in a certain amount of time, but it also increases blood pressure. (1)

X. Issues that Arise (13,12)

Minor changes in blood flow can affect many processes in the body and cause health problems. In minor cases, more stress is simply placed on the heart because more blood has to be pumped throughout the body to compensate for the area that is not receiving enough oxygenated blood. If allowed to progress, however, this leads to high blood pressure- called hypertension- which can cause a plethora of serious health problems. In the U.S., about 30% of the adult population has hypertension, putting them at risk for strokes and heart attacks (13). There are many other common issues that arise due to hypertension (12):

- Coronary artery disease
- Enlarged left heart

- Heart failure
- Dementia
- Mild cognitive impairment
- Kidney failure
- Kidney scarring
- Nerve damage
- Eye blood vessel damage
- Bone loss

Furthermore, hypertension causes vessel walls to weaken in other parts of the body. Sometimes, if the condition is serious enough, a bulge, known as an aneurysm, will form in the vessel. Because pressure is highest in the arteries of the body, aneurysms are more likely to form in the largest arteries of the body, like the aorta, or in areas with high blood flow, like the brain. If the vessel weakens enough, the vessel could rupture and cause internal bleeding (12). Usually, these are life-threatening and will cause death or serious changes that will require months to years of rehabilitation.

Some lifestyle choices can cause damage to the blood vessels and affect blood flow as well. For example, the chemicals that can be found in cigarettes cause the cells of the blood vessels to become inflamed and swell (24). This increases blood pressure as blood vessels constrict, and blood cells are more likely to stick to the vessel walls and form into plaque. Just like with atherosclerosis, this puts excess strain on the heart and can lead to coronary heart disease, stroke, and aneurism. About every one in four deaths in the United States is caused by smoking annually, and the best way to reduce this risk is to quit. Within one year, the risk of having a heart attack already decreases dramatically and will improve one's overall health.

Another disease that can have a major effect on blood flow is diabetes. Because diabetes causes improper absorption of glucose by cells, blood glucose levels are often too high. This causes the blood to become more viscous and "sticky" (25). As one could expect, this causes the heart to work harder, leading to high blood pressure and an increased risk of getting a stroke or heart attack. Similar to with smoking, people can take certain initiatives to prevent the development of diabetes in adults and children in order to increase their overall health and quality of life.

Works Cited

(1) OpenStax. “Anatomy and Physiology.” *Introduction to Sociology – 1st Canadian Edition*, BCcampus, 6 Mar. 2013, opentextbc.ca/anatomyandphysiology/front-matter/preface-2/.

(2) Mada'a, Patrik, and Josef Fontana. “Composition of Blood and Its Functions.” *Functions of Cells and Human Body*, University Development Fund, fblt.cz/en/skripta/v-krev-a-organy-imunitniho-systemu/1-slozeni-krve/.

(3) OpenStax. “Homeostasis.” *Introduction to Sociology – 1st Canadian Edition*, BCcampus, 6 Mar. 2013, opentextbc.ca/anatomyandphysiology/chapter/1-5-homeostasis/.

(4) “Anatomy & Physiology.” *SEER Training:Anatomical Terminology*, training.seer.cancer.gov/anatomy/.

(5) Groopman, Jerome. “The History of Blood.” *The New Yorker*, The New Yorker, 7 Jan. 2019, www.newyorker.com/magazine/2019/01/14/the-history-of-blood.

(6) “Humans and History; the Discovery of Blood. Timeline.” *Timetoast*, www.timetoast.com/timelines/eminent-cairo-is-a-physician-he-is-the-one-who-discovered-the-flow-of-blood-that-goes-to-and-from-the-lungs-also-known-as-pulmonary-circulation.

(7) Kramer, Samantha. “Aztec Human Sacrifice.” *Rise of Civilization*, 25 Apr. 2013, anthropology.msu.edu/anp264-ss13/2013/04/25/aztec-human-sacrifice/.

(8) Klabunde, Richard. “Viscosity of Blood.” *Image for Cardiovascular Physiology Concepts, Richard E Klabunde PhD*, www.cvphysiology.com/Hemodynamics/H011.

(9) T. Doi, et al. . Aviation, Space, and Environmental Medicine, 75, 6 (2004) 500-504.

(10) Greenestone, Gerry. “The History of Bloodletting. BCMJ, Vol. 52, No. 1, January, February, 2010, Page(s) 12-14 - Premise.” *British Colombia Medical Journal*, Jan. 2010, www.bcmj.org/premise/history-bloodletting.

(11) Cohen, Jennie. “A Brief History of Bloodletting.” *History.com*, A&E Television Networks, 30 May 2012, www.history.com/news/a-brief-history-of-bloodletting.

(12) “How High Blood Pressure Can Affect Your Body.” *Mayo Clinic*, Mayo Foundation for Medical Education and Research, 9 Jan. 2019, www.mayoclinic.org/diseases-conditions/high-blood-pressure/in-depth/high-blood-pressure/art-20045868.

(13) “High Blood Pressure Fact Sheet|Data & Statistics|DHDSP|CDC.” *Centers for Disease Control and Prevention*, Centers for Disease Control and Prevention, www.cdc.gov/dhdsp/data_statistics/fact_sheets/fs_bloodpressure.htm.

(14) Majdalani, Joseph. *Brief History of Fluid Mechanics* . majdalani.eng.auburn.edu/courses/02_fluids/handout_f01_history.pdf.

(15) Quinney, D.A. “Daniel Bernoulli and the Making of the Fluid Equation.” *The Mathematics of Diseases*, 25 July 2018, plus.maths.org/content/daniel-bernoulli-and-making-fluid-equation.

(16) Sherwood, Lauralee. “Human Physiology: From Cells to Systems.” *Bionumbers*, Wadsworth Publishing Company, 1997, bionumbers.hms.harvard.edu/bionumber.aspx?id=110559.

(17) Dellinger, Guilhem. “Computational Fluid Dynamics Modeling for the Design of Archimedes Screw Generator.” *NeuroImage*, Academic Press, 17 Nov. 2017, www.sciencedirect.com/science/article/pii/S0960148117310613.

(18) "Isaac Newton." *Biography.com*, A&E Networks Television, 14 Feb. 2019, www.biography.com/people/isaac-newton-9422656.

(19) "Bernoulli's Equation." *Princeton University*, The Trustees of Princeton University, www.princeton.edu/~asmits/Bicycle_web/Bernoulli.html.

(20) "Bernoulli Equation." *Total Internal Reflection*, hyperphysics.phy-astr.gsu.edu/hbase/pber.html.

(21) Sutera, Salvator, and Richard Skalak. "The History of Poiseuille's Law." *Washington.edu*, 1993, depts.washington.edu/jrphys/ph115S13/Pois.pdf.

(22) "Poiseuille's Law: A Historical Background." *Pressure and Blood Flow*, archive.math.arizona.edu/maw1999/blood/poiseuille/history.htm.

(23) Rahman, Shoaibur. "Mathematical Modeling of Blood Flow." *An Introduction to Biometric Recognition - IEEE Journals & Magazine*, Wiley-IEEE Press, 2012, ieeexplore.ieee.org/document/6317446.

(24) "Smoking and Cardiovascular Disease." *Centers for Disease Control and Prevention*, www.cdc.gov/tobacco/data_statistics/sgr/50th-anniversary/pdfs/fs_smoking_CVD_508.pdf.

(25) "Complications of Diabetes." *Government of Western Australia: Department of Health*, www.healthywa.wa.gov.au/Articles/A_E/Complications-of-diabetes.

(26) "Fluids." *MIT*, 2016, web.mit.edu/16.unified/www/FALL/fluids/Lectures/f13.pdf.

(27) Hoffman, J I, and J A Spaan. "Pressure-Flow Relations in Coronary Circulation." *Physiological Reviews*, U.S. National Library of Medicine, Apr. 1990, www.ncbi.nlm.nih.gov/pubmed/2181499.