Modeling Relations Between Red Wine Attributes

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Honors MAT 275

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I. Introduction

Wine is one of the world's oldest alcoholic beverages and remains widely popular to this day. The wine-making process was accidentally discovered from observing grapes spoil and was developed independently into formal production by a multitude of ancient societies, but evidence suggests it was first invented in China (c. 7000 BCE)[1], closely followed by what is now Georgia and Armenia (c. 6000 BCE)[2] and Iran (c. 5000 BCE)[3]. As civilization spread across Asia and Europe, so did the popularity of wine. Its sharp growth in popularity largely had its roots in the perceived medical capabilities of the substance. Hippocrates famously prescribed it as a treatment for a wound as well as for "diuretic, purgative, and sedative purposes."Various other scholars at the time, such as Asclepiades, Celsus, and Dioscorides, praised wine's medical applications for disinfection and anesthesia as well.[4]Because of its numerous uses, around 1000 BCE the Romans made great strides in classification of grapes as well as charting ripening patterns and recognizing soil-type preferences. Up until this point, wine was kept in skins or pottery, but the Romans began to store it in wooden containers and glass bottles, as they noted the difference in taste and maturity.[5]Eventually, sacramental wine became a necessity for the Christian Church, causing its popularity to spread with the religion across the globe. Today, with over 235 million hectoliters[6] produced each year and nearly 18,000[7] wineries worldwide, wine is a major contributor to the world economy, and thus,

[1]Hames, Amy. Alcohol in World History. 2012. London and New York, Routledge Taylor & Francis Group.
[2]Watson, I. (2010, April 20). Unearthing Georgia's wine heritage.
[3]Elsworth, A. (n.d.). The Penn Museum. Retrieved January 29, 2019,
[4]Feher, J, et al. "The cultural history of wine – theoretical background to wine therapy." *Central European Journal of Medicine*. 2007. Retrieved February 16, 2019.
[5]Essays, UK. (November 2018). History of Wine and History of Wine Making Processes.
[6]Vinifera. (n.d.). How many bottles of wine are there in the world? Ask Dr. Vinny.
[7]Simple: Number of Wineries in the World by Country. (2003, March 03).

demand for quality wine continues to climb. The goal of this paper will be to analyze the major components of red wines, which give the beverage its variety of tastes, and to model their interdependencies and correlation to the expertly-determined "quality" of wine.

II. History of Winemaking

Winemaking nowadays is a highly complicated process that requires precise calculation and the consideration of many variables, however it comes from simple origins. Originally, people would simply leave crushed grapes in a bucket for a few daysand thenfilter out the skins, stems, and seeds. What remained was a crude, and often highly alcoholic, beverage that would taste much different to modern wines. During the Classical Era, people would crush grapes, either by foot or with a large stone weight attached to a tree trunk (see *Fig 1*). The juices would flow into a pool where they would be scooped into approximately 100-gallon clay pots along with various spices, such as honey, thyme, and pepper. The juice would then be periodically mixed with broomsticks wrapped in fennel for 6 days to 3 weeks to ferment. The wine would be drinkable for about 10 days after fermentation before it went bad.[8] In the Late Middle Ages, an important stride towards the creation of modern wine was made by using sulfur to stabilize it. The practice was initially illegal because improper use could lead to wine poisoning but was first legislated on the Iberian Peninsula in 1487. Use of sulfur allowed the wine to last much longer than

Fig 1: Grape pressing method using tree trunk and stone weight

[8]Norris, S. (2015, July 21). Winemaking – Do As The Romans Do. Retrieved February 16, 2019

before without the use of potentially poisonous chemical preservatives.[9] As the Industrial Revolution came to be, grape harvesting and fermentation gradually began to be done with machines and winemaking processes evolved into modern methods. These methods are outlined in the following subsections.

a. Grape Cultivation and Harvesting

The *terroir*, or the natural environment where the vineyard resides is incredibly important for the success of a vineyard and is often the first component of a wine's history a sommelier can identify. For optimal cultivation, a vineyard should be in a region that averages 55-70 □ during the growing season, although this varies with different varieties of grapes. Strong winds are potentially threatening to grape yields, as they can often damage the vines or carry disease. Additionally, humidity and rainfall are important to the grape growing process but can also cause issues if not readily monitored. Grapevines need least 20 inches of rain to flourish, which can be easily controlled with irrigation, however, in the event the vine receives too much precipitation, flowering could be stagnated, and the grapes are more vulnerable to fungal growth. If atmospheric moisture levels grow too high, conditions become optimal for certain fungi to grow on and spoil the grapes.Grapes become ripe and ready for harvest about every 165-180 days.[10]When harvesting the grapes, one must consider the timing and the weather, as collecting the grapes too early or late or after a large storm or frost could have a severe impact on the wine.Grape harvests are traditionally done by hand,but many modern vineyards use harvesting machines. These harvesting machines are

[9]Vine to Wine Circle. (2012, May). THE LATE MIDDLE AGES AND THE RENAISSANCE. Retrieved February 16, 2019.
[10]Jones, G. (2015, August 12). Climate, Grapes, and Wine. Retrieved February 16, 2019

naturally much rougher on the grapes due to the violent shaking motion used to jostle the grapes free of the vines. Additionally, if harvesting machines are employed, a layer of CO_2 must be applied to the harvest to protect any broken or damaged grapes from oxidation or bacterial growth.

b. Crushing and Filtration[11]

After harvest, the grapes must be crushed and destemmed to prepare them for fermentation. This process is generally done by machines, however, the traditional method of crushing grapes by foot is still used in some parts of Portugal. Grape destemmer-crushers consist of long horizontal barrel that spins in one direction while rubber paddles in the center rotate the opposite way. Grapes are then fed into the barrel, where the paddles beat the grape until it detaches from the stem and drops though one of many grape-sized holes into the crusher. The stems continue through the barrel until they reach the other end and fall into a collection bin, where they are later used as fertilizer in high pH soils. Once the grapes have been crushed into a 'must,' they are filtered, or 'pressed' through a screen with hole sizes ranging from .45 to 5 microns (micrometers) to remove skins, remaining bits of stem and other solid particles.

c. Fermentation[12]

Once the grapes are filtered,a microscopic fungus called yeast is added, and the mixture sits in large metal vats to ferment.The yeast facilitates the fermentation process by consuming the sugar in the must and converting it to roughly equal quantities of ethanol and carbon dioxide. This process begins when

[11]Quevedo, O. (2011, October 24). How does the grape destemmer crusher work? Retrieved March 3, 2019.
[12] Bailey, R. (2019, January 30). Learn About The 10 Steps of Glycolysis. Retrieved March 4, 2019.

the yeast takes in the glucose from the grapes and starts the process of glycolysis, which produces energy for the yeast, demonstrated by *Fig 2*. During glycolysis, the catalyst NAD^+(nicotinamide adenine dinucleotide) is reduced to NADH, which is used to convert the pyruvate resulting from glycolysis into ethanol via the process outlined in *Fig 3*. Finally, the ethanol is excreted out of the cell, back into the wine-to-be. The entire process takes about three to five days before the yeast begin to die from the high concentration of ethanol.

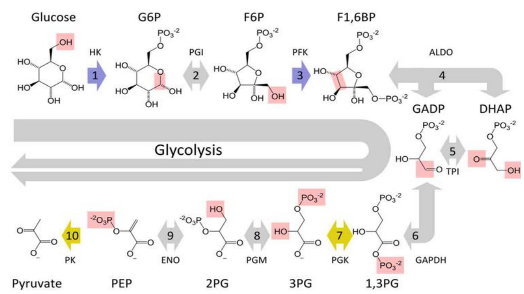

Fig 2: 10 step process of glycolysis

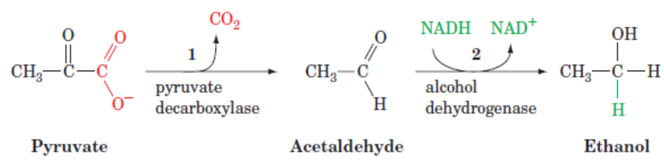

Fig 3: Conversion of pyruvate into ethanol

d. Maturation[13]

After fermentation, the wine is filtered out of the must through a stainless-steel basket and aged in wooden barrels, which allows the wine to extract tannins from the wood to add to those already present from the seeds and skin of the grape, thus enhancing its flavor. While it ages, the wine is occasionally racked or filtered to remove any impurities that have settled to the bottom, such as dead yeast cells or decomposing grape skins. Finally, once it reaches the desired

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[13] Hays, J. (n.d.). Wine Basics and How Wine is Made. Facts and Details.

clarity, the wine is bottled, corked, and possibly allowed to age further in the bottle until it is consumed.

Hundreds of chemical reactions occur during the months or years the wine is left to age, each one altering the wine's final taste in a different way. Among these is the reoxidation of alcohols back to their corresponding aldehydes, reversing step 2 in *Fig 4*. Additionally, various equilibriums are established when carboxylic acids react with the alcohol to form an ester and water and vice versa.[14] However, the most influential reaction that occurs during this time is the depolymerization of tannins. Tannins are flavorful molecules and greatly dictate how the wine will taste. At the beginning of maturation, the wine is full of polymerized tannins, essentially long chains of tannins, but as time progresses, these chains will break up into smaller fragments. Research has suggested that this depolymerization occurs due to the presence of bisulfite, a preservative added during the winemaking process. Bisulfite ions attack the polymer, causing it to break apart and create a sulfonated tannin as a byproduct, as shown in *Fig 4*. This presumption is based off evidence in a 2018 study that

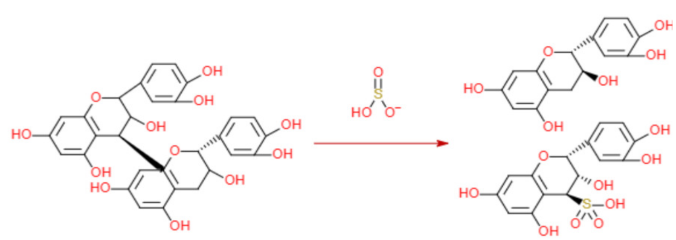

Fig 4: Depolymerization of tannins

[14]Linsenmaier, A. W. (2010). Wine Aging. Retrieved March 5, 2019.

found there to be an inverse relationship between the concentration of bisulfite and the length of tannin polymers in various wines.[15]

III. Component-wise Background

Besides tannins, there are many molecules and qualities that impact a wine's taste and quality. How present a certain compound or trait is in a wine depends on a variety of factors that are enumerated below.

a. Acidity[16]

Acids are major contributors tothe flavor of wine. The acidity of a wine gives it the sour or tart taste, and, without a sufficient amount, it may be considered "flat." Acids in wine can be divided into two major groups: volatile acids and nonvolatile or fixed acids. Fixed acids do not evaporate readily, thus increasing the wine's shelf life, whereas volatile acids cause spoilage in high enough concentrations. Most acids in wine are considered fixed, namely citric, malic, tartaric, and succinic. All of which, except the latter, originate within the grape and are maintained throughout the winemaking process, whereas succinic acid is a product of the yeast during the fermentation process.

b. Residual Sugar[17],[18]

The sugar content of wine dictates its "sweetness" and originates from the grapes it is made of and the processes they undergo. Wine starts out with a high concentration of sugar, most of which gets converted to ethanol via the metabolic

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[15] Lingjun Ma, Aude A. Watrelot, Bennett Addison, and Andrew L. Waterhouse. "Condensed Tannin Reacts with SO2 during Wine Aging, Yielding Flavan-3-ol Sulfonates." J. Agric. Food Chem.66 (35): 9259-9268. Published: May 24, 2018. DOI: 10.1021/acs.jafc.8b01996
[16] The University of California Davis. 2005. What's in Wine?
[17] Szymanski, E. (2012, August 26). What the Heck is Residual Sugar? Palette Press. Retrieved January 29, 2019.
[18] Wu, Y. (2016, January 23). Sweet wines - Methods of production - WSET Level 2. Retrieved January 29, 2019.

processes of yeast during fermentation. Too much residual sugar can not only make a wine too sweet but also creates a prime environment for spoilage microbes, hence it is crucial to control its levels. The residual sugar in a wine can be manipulated in a variety of ways, the first and foremost being fortification. Fortification is when a winemaker adds additional ethanol to the wine, creating a harsh environment for the yeast, thus slowing fermentation, and any microbes that might cause the batch to spoil. If the winemaker wants to make a sweeter wine but without the extra alcohol by volume (ABV), he/she can run the wine through a fine filter to extract the yeast instead. Additionally, the sweetness of the grapes can be altered before they are crushed into the wine by drying or freezing the grapes or allowing a fungus known as Botrytis cinereal to grow on them. These practices result in a much higher residual sugar level and are often used in the making of dessert wines.

c. Chlorides[19]

The chloride content of a wine is synonymous to is salt content, or rather, concentration of free chloride ions in the wine, and thus provides a wine with its saltiness. This factor, like most others, is largely dependent on the climate, topography, and soil of the vineyard and remains relatively constant throughout the winemaking process. The *terroir*is unique to each vineyard, and thus, similar wines are difficult to make, even under the same processes, if the grapes originate from different regions. Chlorides ultimately come from the soil, where it is absorbed through the roots of the vine. Wines generally contain 2 to 4 g/L of

[19]Coli, M. S., Rangel, A. G., Souza, E. S., Oliveira, M. F., &Chiaradia, A. C. (2015). Chloride concentration in red wines: Influence of terroir and grape type. *Food Science and Technology (Campinas),35*(1), 95-99.

various salts of mineral and organic acids, however they can be higher. Some countries have limitations on the amount of chlorides that can be present in marketable wines, for instance Brazilian wine cannot exceed .20 g/L of sodium chloride and Australia caps sodium chloride content at 1 g/L. A 2015 study on the effects of *terroir* on chloride content found that countries with salty irrigation water or brackish terrains, such as Australia and Argentina, produced the saltiest wines.

d. Sulfites/Sulfates[20]

Sulfites, or more specifically compounds which contain SO_3^{2-}, are primarily used in wine as a preservative for their antioxidant and antimicrobial properties. When oxygen dissolves in wine, it can have many adverse effects on its flavor and aroma. For instance, it can react with various phenols, turning the wine to a brownish-red, or it can lead to the creation of acetaldehyde or vinegar, which both give the wine an unpleasant taste. Sulfur dioxide prevents this from occurring by either binding to the precursors of the oxidative reaction to prevent it from reacting with the dissolved oxygen or binding to already oxidized compounds to reverse the reaction. Additionally, SO_3^{2-} can inhibit the enzyme tyrosinase, which often facilitates these oxidative reactions.

Winemakers also use sulfites to selectively inhibit or kill undesirable microbes, such as yeast or bacteria, while leaving the desirable ones unscathed. While sulfites can exhibit antioxidative properties in both free and bound forms, it can only function as an antimicrobial in its free state (i.e. SO_2, HSO_3^-, or SO_3^{2-}).

[20]Henderson, P. (2009, January/February). Sulfur Dioxide Science Behind this Anti-microbial, Anti-oxidant, Wine Additive.

This is because only molecular sulfur dioxide is capable of entering the microbe, and once inside can interfere with the cell's enzyme and protein activity. The availability of these free sulfur dioxide molecules is entirely dependent upon the pH of the wine, and thus, careful manipulation of its acidity is critical. Furthermore, sulfates play a key role in the depolymerization of tannins, which is necessary for the maturation of a wine, as discussed above in section II d.

e. Alcohol[21]

The alcohol content of a wine is the product of the yeasts' respiration process in the fermentation phase of winemaking. As stated previously, the yeast converts the sugars in the must into alcohol, primarily ethanol, via glycolysis, and is then excreted out, back into the fermenting wine. It is for this reason that wines made from grapes with a higher original sugar content will be more alcoholic than others, and thus, riper grapes (which contain more sugar than unripe ones) produce the most alcoholic wines.

IV. MATLAB Analysis

The following MATLAB code develops a model that predicts the quality of a wine, given its fixed acidity (*FA*), volatile acidity (*VA*), citric acid (*CA*), residual sugar (*RS*), chlorides (*Cl*), free/total SO_2 (*F/TSO2*), density (*DEN*), pH (*PH*), sulfate content (*SULF*), and alcohol content (*ALC*). The data used in this model comes from a set of 1,599 anonymous wines, whose quality is scored out of 10.[22] The preliminary model follows the form of *Eq 1*, where *Q* is quality and

[21] Prati, S. (2015, April 17). How does alcohol content affect your wine?
[22] Piyushgoyal443. (2017, March 05). Red Wine Dataset.

a-k are the corresponding weights of each wine component, defined in the code as vector `weights`.

$$a[FA] + b[VA] + c[CA] + d[RS] + e[Cl] + f[FSO2] + g[TSO2] + h[DEN] + i[PH] + j[SULF] + k[ALC] = Q \qquad \textit{Eq 1}$$

The weights were determined by solving for the general form $A\vec{x} = Q$, where $\vec{x}$ is the `weights` vector and*A* is a 1599x11 matrix with each row representing a wine and each column as a wine element. Since *A* is not a square matrix, the equation must be solved with the pseudoinverse by first multiplying both sides by A^T, and then finding the inverse, or $\vec{x} = (A^T * A)^{-1} * A^T * Q$. The matrix of relative error for the prediction of the quality of each wine was found by $relError = \frac{Q_{actual} - Q_{predicted}}{Q_{actual}}$.

This model predicted the quality of various wines with a 2 and 3 norm of relativeerror of .0031854 and .0012500, respectively. The model was then improved on by taking the most influential wine element (*ALC*), determined by the relative size of its weight to others, and adding the square and cubic terms of it to the model. The weights were reevaluated along the framework of *Eq 2*, where *a-m* are now defined by vector `modWeights` in the code.

$$a[FA] + b[VA] + c[CA] + d[RS] + e[Cl] + f[FSO2] + g[TSO2] + h[DEN] + i[PH] + j[SULF] + k[ALC] + l[ALC]^2 + m[ALC]^3 = Q \qquad \textit{Eq 2}$$

The improved model presented slightly more accurate predictions a 2 and 3 norm of relative error of .0031759 and .0012463, respectively.However, as can be seen by the comparative graph in *Fig 5*, both models are relatively accurate for wines with a quality of 5, 6, or 7, but the relative error increases when predicting the higher quality wines. This can be attributed to the large quantity of mid-quality wine in this data set, however the goal of this model is to predict what components make up the highest quality wine, so the data set must be modified to exclude the mid to low quality wines.

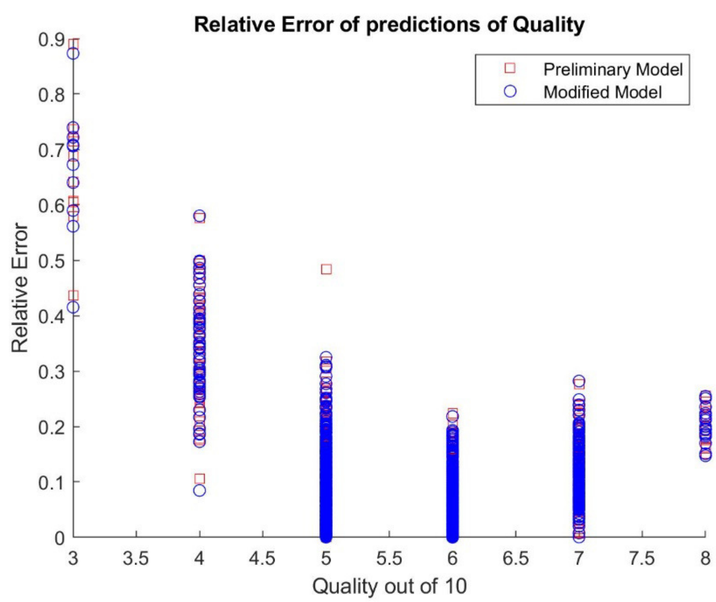

Fig 5: Comparison of relative error of preliminary and modified models

Now, with only the remaining 855 wines of quality greater than 6, the same process as above is performed to obtain a preliminary model of the same form as *Eq 1*. With this new limited data set, the relative weights showed that density, rather than alcohol, was the most prominent factor in determining the quality of a wine, hence, *Eq 2* is modified to the following:

$$a[FA] + b[VA] + c[CA] + d[RS] + e[Cl] + f[FSO2] + g[TSO2] + h[DEN] + i[DEN]^2 + j[DEN]^3 + k[PH] + l[SULF] + m[ALC] = Q \qquad \textit{Eq 3}$$

Unfortunately, this new model offered little improvement of error when predicting wines of high quality, illustrated in *Fig 6*. The 2 and 3 norm of the relative error of the limited model decreased to 0.0022004 and .0008320, respectively.

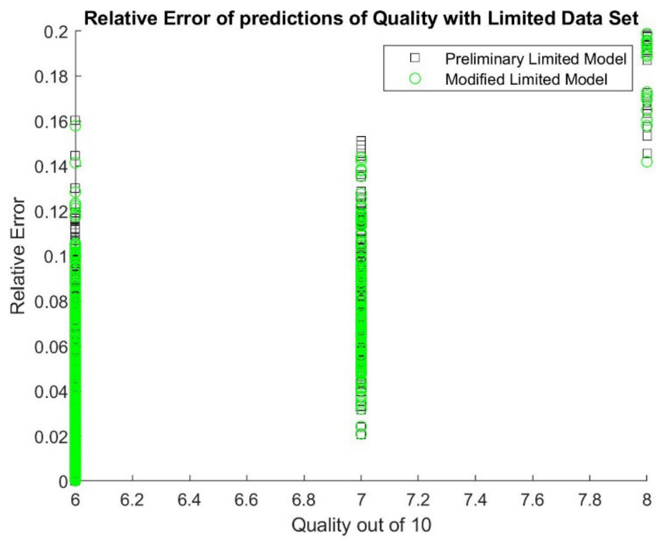

Fig 6: Comparison of relative error of limited preliminary and modified models

V. Conclusion and Further Work

The art of winemaking has been prominent for many millennia and its popularity shows no sign of decreasing in the near future. It is for this reason that the modeling of wine and its components is crucial for satisfying consumers and expanding the industry. Despite its accuracy, the above model is imperfect, especially in predicting high quality wines, and requires further modification, or more specifically, fitting the weights such that each component is broken into linear, quadratic, and cubic parts, similar to what was done with the alcohol content and density components. Additionally, the central assumption of this model is that all factors are independent of one another, which in reality, is far from the case, thus further modeling should also investigate the components' interdependencies. Regardless of its shortcomings, these various models have demonstrated that alcohol content and wine density are the two most influential factors on the quality of a wine, and thus should be given the most attention when crafting wine.

VI. Appendix: MATLAB Code

```
clear
clc

MyData = "RedWineDataSet";
[num,text,raw] = xlsread(MyData);
[row, col] = size(num);

rawFA = zeros(1, 1599);
rawVA = zeros(1, 1599);
rawCA = zeros(1, 1599);
rawRS = zeros(1, 1599);
rawCl = zeros(1, 1599);
rawFSO2 = zeros(1, 1599);
rawTSO2 = zeros(1, 1599);
rawDEN = zeros(1, 1599);
rawPH = zeros(1, 1599);
rawSULF = zeros(1, 1599);
rawALC = zeros(1, 1599);
Q = zeros(1, 1599);

for i = 1:row
rawFA(i) = num(i,2);
rawVA(i) = num(i,3);
rawCA(i) = num(i,4);
rawRS(i) = num(i,5);
rawCl(i) = num(i,6);
   rawFSO2(i) = num(i,7);
   rawTSO2(i) = num(i,8);
rawDEN(i) = num(i,9);
rawPH(i) = num(i,10);
rawSULF(i) = num(i,11);
rawALC(i) = num(i,12);
   Q(i) = num(i,13);
end

rawA = [rawFA' rawVA' rawCA' rawRS' rawCl' rawFSO2' rawTSO2' rawDEN'
rawPH' rawSULF' rawALC'];
```

```
FA = rawFA./max(rawFA);
VA = rawVA./max(rawVA);
CA = rawCA./max(rawCA);
RS = rawRS./max(rawRS);
Cl = rawCl./max(rawCl);
FSO2 = rawFSO2./max(rawFSO2);
TSO2 = rawTSO2./max(rawTSO2);
DEN = rawDEN./max(rawDEN);
PH = rawPH./max(rawPH);
SULF = rawSULF./max(rawSULF);
ALC = rawALC./max(rawALC);

A = [FA' VA' CA' RS' Cl' FSO2' TSO2' DEN' PH' SULF' ALC'];

titles = ["" "Fixed Acidity vs. Quality" "Volatile Acidity vs. Quality"
"Citric Acid vs. Quality" "Residual Sugar vs. Quality" "Chloride Content
vs. Quality" "Free Sulfites vs. Quality" "Total Sulfites vs. Quality"
"Density vs. Quality" "pH vs. Quality" "Sulphates vs. Quality" "Alcohol
Content vs. Quality"];
axislabels = ["" "Fixed Acidity" "Volatile Acidity" "Citric Acid"
"Residual Sugar" "Chloride Content" "Free Sulfites" "Total Sulfites"
"Density (g/cm^3)" "pH" "Sulfates" "Alcohol Content (% v/v)"];
for j = 2:12
   figure(j);
   scatter(num(:,j), Q, 'r.');
   title(titles(j));
xlabel(axislabels(j));
ylabel("Quality out of 10");
   grid on
ylim([1 10]);
end

%%
 weights = (A'*A)^(-1)*A'*Q';
predQ = A*weights;
 Error = Q' - predQ;
relError = Error./Q';


norm1 = norm(relError, 1)/1599;
 norm2 = norm(relError, 2)/1599;
norm3 = norm(relError, 3)/1599;

modA = [FA' VA' CA' RS' Cl' FSO2' TSO2' DEN' PH' SULF' ALC' (ALC').^2
(ALC').^3];
modWeights = (modA'*modA)^(-1)*modA'*Q';
modPredQ = modA*modWeights;
modError = Q' - modPredQ;
modRelError = modError./Q';

figure(1); hold on
scatter(Q, abs(relError),'rs');
scatter(Q, abs(modRelError),'bo');
```

```
title("Relative Error of predictions of Quality");
xlabel("Quality out of 10"); ylabel("Relative Error");
legend("Preliminary Model", "Modified Model");
 hold off

 modNorm1 = norm(modRelError, 1)/1599;
 modNorm2 = norm(modRelError, 2)/1599;
 modNorm3 = norm(modRelError, 3)/1599;

LimitedData = "LimitedRedWineData";
[num2,text2,raw2] = xlsread(LimitedData);
[row2, col2] = size(num2);

limRawFA = zeros(1, 855);
limRawVA = zeros(1, 855);
limRawCA = zeros(1, 855);
limRawRS = zeros(1, 855);
limRawCl = zeros(1, 855);
limRawFSO2 = zeros(1, 855);
limRawTSO2 = zeros(1, 855);
limRawDEN = zeros(1, 855);
limRawPH = zeros(1, 855);
limRawSULF = zeros(1, 855);
limRawALC = zeros(1, 855);
limQ = zeros(1, 855);

for i = 1:row2
limRawFA(i) = num2(i,2);
limRawVA(i) = num2(i,3);
limRawCA(i) = num2(i,4);
limRawRS(i) = num2(i,5);
limRawCl(i) = num2(i,6);
   limRawFSO2(i) = num2(i,7);
   limRawTSO2(i) = num2(i,8);
limRawDEN(i) = num2(i,9);
limRawPH(i) = num2(i,10);
limRawSULF(i) = num2(i,11);
limRawALC(i) = num2(i,12);
limQ(i) = num2(i,13);
end

limRawA = [limRawFA' limRawVA' limRawCA' limRawRS' limRawCl' limRawFSO2'
limRawTSO2' limRawDEN' limRawPH' limRawSULF' limRawALC'];

limFA = limRawFA./max(limRawFA);
limVA = limRawVA./max(limRawVA);
limCA = limRawCA./max(limRawCA);
limRS = limRawRS./max(limRawRS);
limCl = limRawCl./max(limRawCl);
limFSO2 = limRawFSO2./max(limRawFSO2);
limTSO2 = limRawTSO2./max(limRawTSO2);
limDEN = limRawDEN./max(limRawDEN);
limPH = limRawPH./max(limRawPH);
limSULF = limRawSULF./max(limRawSULF);
```

```
limALC = limRawALC./max(limRawALC);

limA = [limFA' limVA' limCA' limRS' limCl' limFSO2' limTSO2' limDEN'
limPH' limSULF' limALC'];
%%
limWeights = (limA'*limA)^(-1)*limA'*limQ';
limPredQ = limA*limWeights;
limError = limQ' - limPredQ;
limRelError = limError./limQ';


 limNorm1 = norm(limRelError, 1)/855;
limNorm2 = norm(limRelError, 2)/855;
 limNorm3 = norm(limRelError, 3)/855;

limModA = [limFA' limVA' limCA' limRS' limCl' limFSO2' limTSO2' limDEN'
(limDEN').^2 (limDEN').^3 limPH' limSULF' limALC'];
limModWeights = (limModA'*limModA)^(-1)*limModA'*limQ';
limModPredQ = limModA*limModWeights;
limModError = limQ' - limModPredQ;
limModRelError = limModError./limQ';

figure(2); hold on
scatter(limQ, abs(limRelError),'ks');
scatter(limQ, abs(limModRelError),'go');
title("Relative Error of predictions of Quality with Limited Data Set");
xlabel("Quality out of 10"); ylabel("Relative Error");
legend("Preliminary Limited Model", "Modified Limited Model");
 hold off

 limModNorm1 = norm(limModRelError, 1)/855;
 limModNorm2 = norm(limModRelError, 2)/855;
 limModNorm3 = norm(limModRelError, 3)/855;
```

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